Design Tools in Didactical Research: Instrumenting the Epistemological and Cognitive Aspects of the Design of Teaching Sequences

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European programs of design research have developed distinctive types of apparatus to structure and support the process of didactical design. This article illustrates how intermediate frameworks and design tools serve to mediate the contribution of grand theories to the design process, by coordinating and contextualizing theoretical insights on the epistemological and cognitive dimensions of a knowledge domain for the particular purposes of designing teaching sequences and studying their operation. The development and analysis of intermediate frameworks and design tools of these types provides a promising approach to establishing a public repertoire of theoretically informed apparatus for didactical design.

Keywords: design research; design tools; didactical analysis; intermediate theory; mathematics education; science education

In recent years interest in design research has spread to mainstream circles in education research (e.g., Kelly, 2003; Sandoval & Bell, 2004). It has been advocated as a mode of educational inquiry that not only provides systematic means for devising and refining novel learning and teaching environments, but couples this with the development of contextualized theories of learning and teaching (Design-Based Research Collective, 2003; van den Akker, Gravemeijer, McKenney, & Nieveen, 2006). Predictably, given the preoccupations of a mainstream audience, discussion has tended to focus on methodological aspects of design research rather than on its distinctive generative element, the design process itself. Talk of "design-based" research (Design-Based Research Collective, 2003) and guidelines that start from "implementing a design" (Collins, Joseph, & Bielaczyc, 2004) focus attention on the analytic process through which an already formed design is progressively refined. This emphasis on "design as implementation" through iterative cycles of revision has been justified on the grounds that the results of "design as intention" are necessarily underdeterminate in their shaping of contextualized practice and correspondingly subject to "lethal mutation" (Collins et al., 2004).

Nevertheless, although iterative refinement of a design through analysis of its implementation is undoubtedly important, the cogency and efficiency with which such revision can be achieved is influenced by the quality of the original design and by the clarity and coherence of the intentions it expresses. Equally, revision of a design often involves taking account of aspects of the working situation that were not recognized or prioritized in the original formulation of the design. Our argument is that the availability of design tools capable of identifying and addressing specific aspects of the situation under design can support both the initial formulation of a design and its subsequent refinement in the light of implementation. In short, producing robust designs and securing well-functioning implementations calls for development of a more systematic apparatus to guide the constructive process through which a design is generated and adapted. This, in turn, is intimately linked with the development of an explicit communal apparatus for design research, instantiated by the recent *Handbook of Design Research Methods in Education* (Kelly, Lesh, & Baek, 2008) and the Design Principles Database (Kali, 2008).

Accordingly, in this article, we seek to contribute to developing a public repertoire of theoretically informed tools for what we shall term *didactical design:* the design of learning environments and teaching sequences informed by close analysis of the specific topic of concern and its framing within a particular subject area. In doing so we draw on relatively longstanding European traditions of didactical research in which the issue of bridging between theoretical principles and design processes has been a central concern. In both mathematics and science education, a major goal is to introduce students to disciplinary concepts that have been refined and formalized so that they differ markedly from the more informal ideas current in everyday thought and already available to students. A key aim of didactical design is to devise teaching sequences that not only are suitable for widespread use in ordinary classroom circumstances but are sufficiently comprehensive and robust to achieve their intended effects in a reliable way. The promise of design research is in providing more powerful and direct means of developing such teaching sequences and associated theorizations that can assist local adaptation of these sequences to take account of crucial contextual features.

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There are some significant parallels with North American learning-sciences research aimed at novel treatment of curricular areas, notably a common inheritance from clinical interview studies and teaching experiments (Confrey, 2006). In particular, we should emphasize that the design of what are conventionally termed *teaching sequences* focuses on supporting learners' construction of disciplinary concepts. However, there are also important contrasts that reflect the institutional context in which mathematics and science teaching take place in European countries where national curricula are tightly specified and more strongly focused on substantive knowledge than on syntactic knowledge (Schwab, 1978). In comparison, recent work by North American learning-sciences researchers typically describes features of "learning environments" intended to support the longer term development of learners' engagement in disciplinary practices, including reasoning about evidence and explanation, representing and communicating information, evaluating knowledge claims, and building and refining models and theories (e.g., authentic practices, Edelson & Reiser, 2006; model-based reasoning, Lehrer & Schauble, 2006; project-based learning, Krajcik & Blumenfeld, 2006). A distinctive characteristic, then, of the European tradition of didactical research is that the teaching and learning of specific topics are addressed at a fine grain size over relatively short time sequences, best characterized in terms of number of hours of teaching, rather than weeks or months or years (Méheut & Psillos, 2004).

Traditionally the process of didactical design has largely been informed by the professional knowledge, often tacit, of the designers. However, the design process may draw upon other kinds of knowledge, not least "grand theories" (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003; Skinner, 1985): theories general in scope and correspondingly abstract in form; notably theories of human development and learning, of the epistemology of the discipline, or of the process of instruction. Moreover, it is not unusual to find distinct grand theories of these types informing different aspects of the design process. This has led designers to create more specific frameworks, intermediary between grand theory and the process of design. Such frameworks extract, coordinate, and contextualize those aspects of several grand theories that are pertinent to developing, analyzing, and evaluating teaching designs. Thus we share with some North American design researchers a concern with developing intermediate theories and frameworks "accountable to the activity of design" (Cobb et al., 2003).

This article will focus specifically on approaches to analyzing the epistemological and cognitive dimensions of a knowledge domain so as to inform both design of a teaching sequence and study of its operation. Through comparing and contrasting approaches, we will develop more overarching ideas about the relations between grand theories, intermediate frameworks, and design tools. The particular didactical frameworks that we will examine have been developed, largely independently by different teams, to inform the design of teaching sequences in either school mathematics or science. Specifically, these comprise:

- The *Theory of Didactical Situations* and its associated tools of *Adidactical Situation* and *Didactical Variables*, concerned with mediating the construction of new mathematical knowledge through independent problem solving by students;
- The *Two Worlds* framework and its associated tools of *Knowledge Distance* and *Modeling Relations*, concerned with planning knowledge development in the physical sciences and with identifying relations that need to be made explicit within and between theoretical and empirical domains;
- The *Social Constructivist Perspective on Science Learning in Formal Settings* and its associated tools of *Learning Demand* and *Communicative Approach,* concerned with identifying detailed content-specific teaching goals and with choosing forms of classroom discourse well matched to particular teaching and learning purposes.

An Example of Didactical Design of a Teaching Sequence

This section provides an overview of the process of didactical design, illustrating how epistemological and cognitive analyses underpin design choices. We will show how ideas from grand theories lie behind key elements of the intermediate framework and how these are associated with the design tools used to craft teaching sequences. To illustrate this, we will first introduce an intermediate framework specifically developed to steer the didactical design of teaching sequences in mathematics, the Theory of Didactical Situations (TDS); then we will show how a particular teaching sequence was designed under its guidance.

The Theory of Didactical Situations as an Intermediate Framework for Design

Over the course of its development, TDS has drawn on a wide range of theoretical ideas. According to its originator, Brousseau (1997), TDS was strongly shaped by Piaget's theorization of cognitive development as a process of constructive adaptation and was refined in the light of Bachelard's theorization of knowledge growth as encountering intrinsic obstacles. We will show how the formative influence of these grand theories on TDS was expressed in selective appropriation of particular components from them rather than any wholesale synthesis. TDS developed, therefore, as an intermediate framework in which components appropriated from grand theories were combined, elaborated, and refined to produce an explicit didactical apparatus for the design of teaching sequences, an apparatus that—although TDS itself does not use such a term—includes identifiable design tools (Table 1).

Thus Piaget's theory of "equilibration" was a crucial source for the idea of "a-didactical" adaptation in which students construct new knowledge through becoming directly engaged in solving a novel type of problem, refining their concepts and strategies in the light of feedback from a (material and social) milieu (Brousseau, 1997, pp. 64, 147). And although Piagetian grand theory already incorporated the idea that cognitive development depends on reorganization of thinking, the TDS framework extended it by appropriating the further idea of "epistemological obstacle" from Bachelard's grand theory of knowledge growth in the physical sciences, to highlight the particular significance in learning mathematics of unavoidable and ultimately productive "obstacles . . . from which one neither can nor should escape, because of their formative role in the knowledge being sought" (p. 87). In order, then, to identify and analyze these crucial epistemological obstacles within a particular mathematical domain,

Table 1
Grand Theories, Intermediate Framework, and Design Tools in Brousseau's Approach to the Design of Teaching Sequences in Mathematics

Essentials of Design Approach	Design Based on Adidactical Adaptation
Grand theories	Piagetian perspective on knowledge construction; Bachelardian concept of epistemological obstacle
Intermediate theoretical framework	Theory of Didactical Situations
Design tools	Adidactical Situation; Didactical Variables
Key summary references for further details about the framework and tools	Brousseau, 1997; Brousseau, Brousseau, & Warfield, 2008

TDS triangulates the findings of psychologically informed studies and historically informed analyses.

Central to the design apparatus of TDS is what we will term the Adidactical Situation tool. Here *situation* refers to an ensemble of problem-solving task and task environment designed to evoke a particular form of adidactical adaptation on the part of students, intended to help them construct some specific new knowledge. The tool specifies the necessary components for creation of such an "adidactical situation," guided by the theorization that TDS provides of their functioning as a system: the problem to be posed, the conditions under which it is to be solved, and the expected progression toward a strategy that is both valid and efficient. This includes the process of "devolution" intended to lead students to directly experience the mathematical problem as such and the creation of a (material and social) "milieu" that provides students with feedback conducive to the evolution of their strategies.

In the course of its development as an intermediate framework, TDS has responded to weaknesses that emerged in its functioning that reflect limitations of the guiding grand theory. In particular, an early assumption was that teaching sequences could be organized around adidactical situations alone. As Brousseau (1997) recounts, "It took us some time to realize that [teachers] really needed to do [other] things, for reasons that had to be understood" (p. 236). This provoked modification of the intermediate TDS framework and critique of the grand Piagetian theory as neglecting this important dimension: "The [inescapable] intervention of the (mathematical) culture through the medium of the teacher" (p. 110). It led to the incorporation of a further stage of "institutionalization" in which the knowledge that students have developed from an adidactical situation undergoes a process of socialization and codification. Equally, in adapting the Bachelardian idea of an epistemological obstacle intrinsic to the development of mathematical thinking, TDS found it necessary to differentiate this from a didactical obstacle originating contingently in a particular way of organizing the teaching of mathematics. Thus these types of refinement of the TDS framework have also produced a re-equilibration of its relationship to grand theory. Nowadays TDS has become sufficiently well developed and recognized as an intermediate theory in its own right, specifically adapted to the needs of didactical design, to make reference back to Piagetian and Bachelardian grand theories rare.

The Design of a Teaching Sequence in Mathematics

The treatment of decimal numbers at the primary-school level provides a very good example of a long-term teaching sequence designed on the basis of this didactical theory. The resulting sequence has been replicated more than 10 times, consistently producing the predicted patterns of student response and outcome (Brousseau, 1997, p. 184). We will explain how the design choices derive from theoretical analysis, brought to bear through the Adidactical Situation tool.

In TDS, the guiding epistemological hypothesis is that a mathematical concept takes its meaning from problems to which it brings an optimal solution, in particular an economical one. For instance, comparing fractions can be done especially efficiently when they are written in a decimal form such as 0.625 and 0.650 rather than in a rational form such as $^5/_8$ and $^{13}/_{20}$. The corresponding cognitive hypothesis within TDS is that learning results from students adapting their mathematical thinking in response to some new situation where their existing knowledge does not support an efficient solution strategy. The means of solution that students find it necessary to devise serve as the source of new mathematical knowledge. Epistemological and cognitive considerations are, of course, not independent: The aim is to identify the conditions for a planned process of learning through which students construct and use those features of decimal numbers that the epistemological analysis has identified as constituting the concept.

A distinction can be drawn between choices relating to the "framing" of the teaching sequence—the selection of mathematical concepts and the formulation and representation of those concepts in terms of the problems that constitute the core of the desired learning—and choices relating to the "staging" of the teaching sequence—the organization of learning objectives related to the mathematical content in question in relation to a coherent sequence of lessons incorporating problem situations capable of stimulating the intended learning.

Framing a teaching sequence. The teaching sequence for decimal numbers aims to support students in constructing a multifaceted meaning of the concept of decimal number that they are able to use adequately and efficiently across different kinds of problem. Drawing both on psychologically oriented studies of the development of mathematical thinking and on historically informed analysis of the evolution of mathematical ideas, Brousseau set out to identify a suitable system of problem situations capable of supporting the construction of new solution strategies by the students and their transformation into new mathematical knowledge. Initially, he examined how decimal numbers had evolved within the wider field of mathematics so as to identify the key mathematical problems that gave rise to decimal numbers and to clarify the relationships between decimal numbers and other types of numbers, especially rational number, typically expressed in the form *a/b*.

Decimal numbers as measures arise from splitting some unit; as such they are linked to notions of ratio. However, these decimal measures represent a very particular type of ratio of whole numbers in which powers of 10 are privileged. Indeed, the epistemological analysis identified two fundamentally different overarching approaches to the teaching of decimal numbers: Either decimal numbers are presented first as an extension of whole numbers, and then extended further to encompass all rational numbers; or rational numbers are introduced from the start as an extension of whole numbers, with decimal numbers following as a particular type of rational number. To inform his choice between these two approaches, Brousseau investigated past curricula and their impact on students' conceptions as shown by research.

In France, the primacy of the metric system had led to the first approach being prominent in past curricula. Indeed, the introduction of the metric system in late-18th-century France was part of a grand post-Revolutionary program to develop universal standards, civic rationality, and public education (Lacombe, 1979). In the traditional didactical sequencing, then, decimal numbers were introduced to students as a means of measuring magnitudes (such as length or weight). The approved way of representing—in particular tabulating—these measures often led students to conceive a decimal number as an entity made up of two whole numbers: for example, 15.64 meters could be read as 15 meters and 64 centimeters. This conception of decimal numbers as a pair of whole numbers proved to be particularly persistent over time—it could even be detected among university students—causing errors in calculation with decimal numbers or in comparison of decimal numbers (Sackur-Grisvard & Léonard, 1985). Moreover, conceiving decimal numbers as a pair of whole numbers does not encompass the notion of the successive splitting up of the unit—and consequently of ratio—and may even hinder the construction of such a meaning.

The teaching sequence developed by Brousseau is based on the second approach. Here, the epistemological rationale is to introduce decimal numbers as economical tools through which comparing, adding, and subtracting fractions can be done more quickly and with fewer errors, and particular types of problem solved—such as finding a new fraction lying between two given fractions. At the same time, the cognitive rationale is to avoid students constructing (as already outlined) the conceptions of decimal numbers associated with the first approach that constitute obstacles (of didactical origin) to the notion of ratio. In sum, then, the main thread of the teaching process that Brousseau devised as a result of these analyses involves constructing rational numbers as tools for measuring, and then decimal numbers as tools for approximating rational numbers. The final part of the teaching sequence focuses on rational numbers as operators, culminating in construction of the product of two rational numbers in terms of the composition of two mappings.

Staging a teaching sequence. Once these fundamental choices have been made, determining the overarching structure of the teaching sequence, its staging must be designed in greater detail. Following from the theoretical assumption that learning results from adaptation to problematic situations, the teaching sequence is devised around a succession of problems forming a learning progression. The question proposed to students at each new stage in this progression arises from problems encountered in solutions at the previous stage or from the consequences or developments of these solutions. This succession does not depend only on the prior epistemological analysis but also on considerations of the local organization and functioning of particular situations within the teaching sequence. Indeed, the final teaching process results from a dialectic, between the original design and its realization in the classroom, in what is usually an iterative process of development.

The design of the particular tasks, and the way in which they are to be presented to students, is guided by the Adidactical Situation tool. Although an adidactical situation is designed to condition the construction of some specific new knowledge by students, it must be experienced by students not as a matter of learning some ready-made result, but rather as one of resolving a genuinely problematic state of affairs with whatever knowledge they already have available. In particular, an adidactical situation depends on the problem being such that some starting strategy is available for students, but one that turns out to be unsatisfactory in some way. The ideal is that students, as a result of observing the inadequacy of their strategy, will be motivated to look for others and that this will lead them to devise solution strategies that provide a basis for constructing the intended new knowledge.

Thus it is of crucial importance that students become aware of the inadequacy of their tentative solutions and that they receive information from the situation to enable them to move forward in developing a more powerful solution. The notion of "milieu" has been developed within TDS to refer to that component of the situation that offers possibilities of interaction to students, providing means of gaining feedback to validate or invalidate their solution strategies. Particularly where younger students are concerned, the milieu is often designed to capitalize on a context with which students are already familiar. This familiarity guides the opening exchanges between situation and students. Changing the context for each particular situation in the teaching sequence is impractical, as it would require students to spend time coming to terms with a new context on each occasion. Moreover, if the same context can be maintained, students' greater familiarity with it facilitates the exchanges. Finding a suitable context, capable of serving over several sessions, is therefore a critical issue in the staging phase of a teaching process.

Designing an adidactical situation. Here we give an account of the design of one of the adidactical situations, which forms part of Brousseau's teaching sequence for the construction of rational numbers as linear mappings. Its context—the enlargement of a shape puzzle—is much more than an ingenious choice: It is the result of an extended and systematic process of analysis and refinement. First, this is a context that appeals to ideas and activities likely to be familiar to students. Most have already played with shape puzzles and know that solving them means fitting all the pieces together without gaps or overlaps, and they have experienced enlargement early in primary school in the course of drawing. In addition, the physical character of the puzzle contributes to the availability of visual feedback. All of these features serve to assist "devolution" of the problem in the classroom: the process through which students themselves are brought to consider the problem an authentic one and engage with it.

Moreover, an adidactical situation is not just a task given to students: A specific social organization of student work has to be planned, including collective phases involving the whole class. In

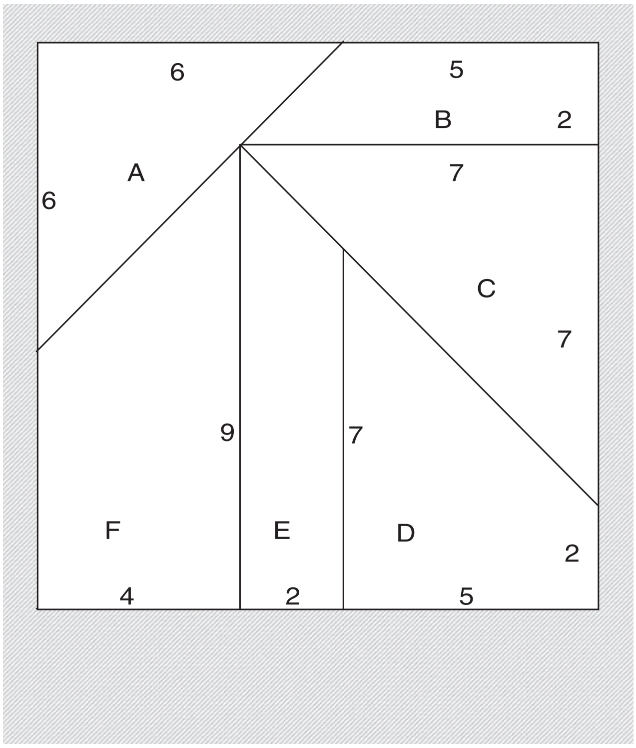

FIGURE 1. *The puzzle to be enlarged in the illustrative adidactical situation.*

this situation, students are initially expected to work in groups of four or five. Each group is given the pieces of a cardboard puzzle to fit together (as shown in Figure 1). They can do this without difficulty. Then they are asked to make a larger but otherwise identical puzzle: specifically, the edge that measures 4 cm in the original puzzle should measure 7 cm in the larger one. Each student in a group has to work on at least one piece. The expectation is that they will use geometric invariants when forming the new pieces and additive reasoning when determining their edges, so that they always add 3 cm to the original length. But when they try to put together the new pieces, it will become clear that they do not fit properly. The students may think that some of them have simply been careless. But if they try again, the result remains the same: The pieces never fit together perfectly.

This situation was created expressly with the intention of addressing a crucial epistemological obstacle. It seeks to invalidate an additive model of the scaling operation through providing students with strong feedback that convinces them that their solution is wrong. The visual evidence alone should be sufficient to show that the pieces of an additively enlarged puzzle will not fit together. This feedback is effective because it has been designed to be interpretable using mathematical knowledge that students already have and because critical characteristics of the situation have been carefully engineered. But this does not mean that students will automatically adopt a multiplicative model in which lengths are enlarged by a factor of $^7/_4$. Such a model will only be constructed after experience of several problem situations has fostered the idea of the linearity of enlargement. This normally appears after other incorrect models have been proposed, such as the generalization arising from doubling 4 cm to get 8 cm then subtracting 1 cm to give 7 cm. The pieces made according to this rule almost fit together, and again students think that they have just not been precise enough.

At this stage, however, the pragmatic feedback offered by the material milieu is no longer sufficient. It becomes necessary for the teacher to offer *intellectual* feedback drawing on the knowledge available to the students. The original square needs to be transformed into a larger square. By focusing on the process of applying the doubling-and-subtracting rule to the differing partitions of the sides of the original square and adding the results, it becomes clear that squareness is not preserved. It is usually only after students have dismissed computationally simple formulations of this type that they find a strategy based on the linearity of the transformation for sums of lengths: For example, the image of 6 cm is the image of 2 cm added to the image of 4 cm. But the choice of lengths like 5 cm in the original puzzle obliges students to eventually resort to the image of 1 cm. The role of this unit ratio may be discovered by students or prompted by interventions from the teacher: As soon as students have found the enlarged size of 1 cm, they can handle any length along the border of the puzzle.

Identifying and optimizing didactical variables. However, neither grand nor intermediate theory is able to inform every aspect of a particular design needed to engineer the desired pattern of response to a situation. To anticipate during the process of design what further factors may prove crucial, and equally to manage the control of unexpected factors that emerge during trialing, a further design heuristic is used. The Didactical Variables tool provides a framework for making careful choices concerning characteristics of the original puzzle and the enlargement, specifically the choice of

- the shape of the pieces,
- the lengths of the sides of each piece along the border of the puzzle, and
- the ratio of the enlargement.

Each of these choices was based on analysis of students' prior knowledge. It was expected that students would understand that enlargement preserves shape, at least for familiar shapes. Therefore the puzzle was chosen to be square in shape and its pieces to be triangular and trapezoidal in shape and to feature right angles. In order to avoid making enlargement calculations too complex, and to afford development of the unit ratio strategy, the lengths given to sides (particularly along the border of the puzzle) were chosen to be whole numbers. And because the goal was to introduce rational numbers as operators of a linear mapping, the enlargement ratio was chosen to be a fraction. As well as the shape of the pieces, the lengths of the sides, and the ratio of enlargement, two other features of the situation are critical:

- The number of pieces along each side of the puzzle differs.
- Each piece is made by a different student.

The first of these conditions ensures that use of the additive model results in a striking anomaly when the equal borders of the original puzzle (as shown in Figure 1; e.g., those on the left and right sides of the puzzle where $6 + 5 = 2 + 7 + 2$) are "enlarged" to produce unequal borders (as when 3 is added to each component length to produce $9 + 8 \neq 5 + 10 + 5$). The second condition

Table 2

Grand Theories, Intermediate Framework, and Design Tools in the Lyon and Leeds Approaches to the Design of Teaching Sequences in Science

Essentials of Design Approach	Design Based on Modeling (Lyon Group)	Design Based on Social Constructivism (Leeds Group)
Grand theories	Epistemology of science (Bachelard, Hacking); Vygotskian perspective on learning	Realist ontology (Harré); Sociocultural account of meaning making on the social plane (Vygotsky, Bakhtin, Wertsch)
Intermediate theoretical framework	Two Worlds	Social constructivist perspective on science learning in formal settings
Design tools	Knowledge Distance; Modeling Relations	Learning Demand; Communicative Approach
Key summary references for further details about the frameworks and tools	Buty, Tiberghien, & Le Maréchal, 2004; Tiberghien, 1996, 2000	Leach & Scott, 2002, 2003; Leach, Hind, Lewis, & Scott, 2006; Mortimer & Scott, 2003; Scott, Mortimer, & Aguiar, 2006

prevents students from simply drawing a square of side 7 cm on the cardboard and then cutting each piece of the puzzle from this, adjusting the lengths of their sides.

Choices of this type can be thought of as a matter of specifying the values of variables where what might appear to be inconsequential choices significantly affect students' solving strategies. These are called *didactical variables* because they act as key levers to precipitate and manage the unfolding of the expected trajectory of learning. Identifying such variables starts from analysis of the knowledge available to students. Observation of how situations play out with students in the classroom may reveal further variables not identified through the prior analysis. As in the "microcycles" described by Gravemeijer and Cobb (2006), conjectures about the impact of didactical variables on students' mental activities are tested in instruction, potentially leading to their refinement. This process contributes to specifying conditions that have a critical influence on the learning potential of a situation. Essentially, then, detailed design of an adidactical situation and analysis of the operation of its milieu depend on the identification and optimization of such variables, making the Didactical Variables heuristic an important design tool.

Intermediate Frameworks and Design Tools as Instruments to Guide Theoretically Informed Local Decisions About the Design of Teaching

As noted earlier, TDS has become sufficiently well developed and recognized as an intermediate theory in its own right for it to have become rare for reference to be made back to the "grander" theories that shaped its development. By contrast, the examples to be presented in this section illustrate, in the absence of an overarching intermediate theory, a more explicit interplay between grand theories, intermediate frameworks, and design tools in designing teaching sequences in the physical sciences.

These examples arise from the independent work of two groups (based in Lyon, France, and Leeds, United Kingdom). However, as shown in Table 2, both groups draw upon Vygotsky's sociocultural view of learning. This view suggests that higher mental functioning (such as conceptual understanding) is underpinned by social language and other semiotic mechanisms (Vygotsky, 1978). These provide the means for concepts to be talked through between people on the social (or intermental) plane—be these scientific concepts or the concepts that are used in everyday communication. In the process of *internalization* (Vygotsky, 1987), individuals appropriate and become able to use for themselves (on the intramental plane) conceptual tools first encountered on the social plane. Linguistic and other semiotic mediation plays a crucial role in this process. Central to this view is the continuity between language and thought. It is not the case that language offers some neutral means for communicating personally and internally generated thoughts: Language provides the very tools through which those thoughts are first encountered on the social plane and then appropriated on the intramental plane. We will illustrate how this body of theory has informed the design process—at both large and fine grain size—through the discussion of examples.

Given this common theoretical influence, it is not surprising that the design problems addressed by both groups, and indeed many of the design solutions that have been developed, share important similarities, although, as we shall see, there are significant differences in the ways in which the Vygotskian grand theory has been used in the design process. For this article we have selected some examples (summarized in Table 2) to illustrate how intermediate frameworks are used to make links between grand theories and, through the use of associated design tools, to devise teaching sequences.

As with TDS, the intermediate frameworks developed by both groups establish a relationship between epistemology, learning, and teaching in a form tailored to the design and analysis of domain-specific teaching sequences (i.e., the nature of the content is treated as an important influence on how that content is best taught and learned). Both frameworks draw mainly on grand theories about learning and epistemology to underpin their proposed framing and staging of teaching sequences, rather than on grand theories about teaching. Equally, as the example of the previous section illustrates, an important feature of the design process is that many key decisions are at a very fine grain size, relating to specific aspects of the content to be taught or the teaching approach (Leach & Scott, 2008; Tiberghien, 1996; Tiberghien, Gaidioz, & Vince, in press). Although intermediate frameworks derive from grand theories' general orientations about productive features of learning environments, they are not wholly capable of guiding decisions about the design of teaching at this fine grain size. Accordingly, design tools have been developed to bring insights from the intermediate framework to bear on the fine detail of the design process.

Two Worlds as an Intermediate Framework for Design (Lyon Group)

Two Worlds was developed as an intermediate framework to inform the design of teaching sequences in physics for the upper secondary school, taking two fundamental orientations from grand theories. First, drawing on epistemological theorization of the experimental sciences (and particularly physics), modeling is treated as a foundation for scientific knowledge (Bachelard, 1979; Hacking, 2005/1983). Second, drawing on Vygotskian theorization of learning, the physics classroom is viewed as a place where students are invited to participate in an educational community where one of the roles of the teacher is to portray some of the knowledge and practices of professional physics communities (Tiberghien, 1996). As adapted to the upper secondary school, the epistemological hypothesis underpinning the Two Worlds framework is that processes of modeling play a central part in understanding physics by relating descriptions of objects and events in the material world to the world of theories/models. Everyday knowledge and physics knowledge each offer ideas and languages (which overlap to a degree) for *describing* objects/events in the material world; these are linked via modeling processes to distinctive theories/models for *interpreting, predicting,* or *explaining* events in the material world. However, even when such objects and events are easy to perceive, this does not imply that scientific descriptions or interpretations of them are similar to everyday ones. The design process, therefore, takes particular account of students' prior knowledge, following the hypothesis that whenever a person or a group explains, interprets, or makes a prediction about the material world, some form of modeling activity is involved (Tiberghien, 2000) and that this is the case in everyday life just as in the domain of physics.

The Two Worlds framework was developed to inform the design of teaching sequences that take account of students' everyday knowledge and its relationship to physics knowledge. Hence it makes a double categorization of knowledge into *everyday knowledge* and *physics knowledge*. Within each of these categories, theories/models are distinguished from descriptions of objects/events in the material world (Figure 2). As the diagram indicates, there can be some overlap between physics knowledge and everyday knowledge. For example, intuitive knowledge about *speed* has much in common with the notion of *velocity* in physics—although velocity has a directional quality that distinguishes it from the everyday idea of speed. The distinction between the world of theories/models and the world of objects/events serves to make explicit the modeling processes that establish relationships between them. For example, physics knowledge distinguishes the theoretical concept of force from the material event of one object acting on another, and likewise the theoretical concepts of trajectory, velocity, and change in velocity (acceleration) from observable changes in movement. Through linking these concepts, physics knowledge creates a theoretical model capable of predicting observable motion (such as the way in which an object will fall when it is dropped by a moving cyclist).

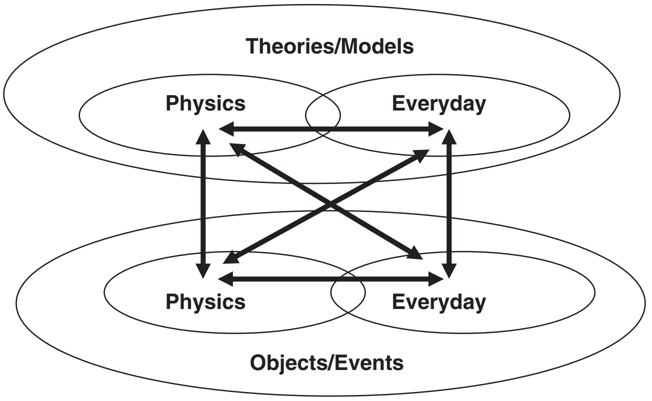

FIGURE 2. *Double categorization into everyday knowledge and physics knowledge of the Two Worlds of theories/models and described objects/events as used at the secondary school level. The arrows show the possible interpretative, predictive, or explanatory links that can be established.*

The Two Worlds framework assumes that relationships between all categories of knowledge are possible, as indicated by the arrows in the diagram. In particular, students can consider relationships between their everyday descriptions of objects/events and theoretical elements of physics knowledge that they have learned. Equally, students can examine relationships between their everyday theories about the behavior of the material world and the physics theory presented during teaching.

For the purpose of informing the design of physics teaching, two particular design tools have been developed from the Two Worlds framework. These tools are complementary: the Knowledge Distance tool guides the framing and sequencing of the teaching content, whereas the Modeling Relations tool guides the design of specific teaching activities at a finer grain size. Both design tools are informed more strongly by grand theory of epistemology (Hacking, 1983/2005) than by Vygotskian grand theory of learning. However, both reflect the core Vygotskian assumption that knowledge exists on two planes and that all knowledge has its origins on the social plane. Likewise, both tools are informed by the Vygotskian notion of the zone of proximal development. The assumption is that when students encounter new knowledge, they make sense of it in terms of existing knowledge. Consequently, care must be taken to ensure that the distance between these knowledge states is not too great; the gap must be capable of being bridged by students with help from the teacher and/or other students. An inevitable consequence of this assumption is that it is not possible to introduce the scientific meaning of a concept to students all in one step; it is only possible to introduce partial components. Both design tools therefore require the designer to "break down" knowledge into smaller components for the purpose of design.

The Knowledge Distance tool. The Knowledge Distance tool makes explicit the difference between the knowledge to be taught and students' existing knowledge, as analyzed in terms of modeling (Buty, Tiberghien, & Le Maréchal, 2004), providing a detailed mapping of the distance between the two. This tool is

Table 3
Using the Knowledge Distance Tool to Analyze the Relationship Between Action and Force for Teaching Purposes at Grade 10 Level

Components of the Two Worlds Framework	Students' Existing Physics Knowledge	Students' Existing Everyday Knowledge	Physics Knowledge to Be Taught in the Sequence
Theory/model	Velocity. Uniform and nonuniform motion. *These concepts are taught just before the introduction of force (Grade 10); assessments suggest that they are at least partially understood by students.*	Motion requires force (in its everyday meaning). *Studies show that most students consider that force is the cause of motion—which is characterized by the speed.*	Force is exerted by one system on another and represented by a vector. Laws of mechanics. *This analysis of the curriculum is based on the modeling approach.*
Relationship between the two worlds (theory/model, objects/events)	An object is represented by a point; its trajectory by a line; and its mean velocity by the ratio of the distance between two positions of the point to the time taken to travel between them. *Again, assessments suggest that students readily grasp the notion of mean velocity.*	Some changes in the motion of an object require an action or a force (in its everyday meaning). Situations in which there is no motion are not normally related to force (in its everyday meaning) or action.	Account of the relation between force and action covering cases with and without motion. Where there is contact between objects, there is also a force between the objects. The Earth exerts a force on objects. *In the modeling approach force is an abstract concept used to model action, which is a term used to describe an event in which one object acts on another (even if there is no motion).*
Objects/events	Situations with different motions. *In physics, motionless situations are not treated as special cases; rather, they are particular situations where v = 0 on a continuum.*	Situations involving separate types of motion, and motionlessness, treated in different terms. *In everyday situations, the motionless situation is usually treated as completely different from those where there is motion.*	Action of one object on another (with and without motion) and the reverse. Action is due to contact between objects, or action is at a distance (in cases such as gravitational attraction).

used to fill out a general statement of the content to be covered in a curriculum by identifying the specific components of that knowledge that students need to understand.

The use of the Knowledge Distance tool is exemplified in Table 3, which shows how it informed the design of a teaching sequence on mechanics. Using the Two Worlds framework, a double classification of students' knowledge was made in terms of the two dimensions: theory/model versus object/event; and physics knowledge versus everyday knowledge (Figure 2). Insights about students' existing knowledge were drawn from diverse sources, including assessments carried out by the teacher following earlier teaching, and synthesis of findings from the research literature. The final column of the table highlights the physics knowledge that students are intended to learn as a result of teaching, where this is particularly different from students' existing knowledge. Drawing upon all of these sources, the designer identified those relations between the world of theories/models and the world of objects/events that would not already be known by students, so as to make these the focus of the teaching sequence. In the example shown in Table 3, the concern is to establish a relationship between force and action that covers all cases regardless of whether or not there is motion. Because the sequence aims to help students come to distinguish between their everyday notion of force and the physics concept of force, it begins by focusing on action in the world of objects/events. Starting from the idea of action provides a suitable language with which to describe material situations in anticipation of introducing the physics theory/model. In physics, force is treated as an abstract concept in the world of theories/models that has then to be explicitly related, through the idea of action in the world of objects/events, to motion (and motionlessness).

The Modeling Relations tool. The second tool makes explicit the kinds of relations that teaching should lead students to establish within and between worlds. Table 4 summarizes four different types of relation on which an activity may focus. A complete teaching sequence to help students construct a new system of concepts should include activities covering the full range of

Table 4

Types of Relation Within and Between the Two Worlds That Are Distinguished by the Modeling Relations Tool, Illustrated in Terms of the Topic of Force and Action at Grade 10 Level

Type of Relation	Character of Relations That Are the Focus of Attention	Illustration of Intended Evolution in Students' Comprehension of Such Relations
Type 1	Relations within the world of objects and events	Evolution from the everyday meaning of action to the physics meaning: i.e., from the idea that there is action only when some change happens (everyday meaning) to the idea that there is action even if there is no apparent change, as when motionless objects are in contact (physics meaning).
Type 2	Relations from the world of objects and events to that of theories and models	Evolution from the physics meaning of action and everyday knowledge of force to construction of the relation between "action" events within a physics description and "force" concepts within a physics model: i.e., if two objects are in contact with one another, then the system representing one object exerts a force on the system representing the other.
Type 3	Relations from the world of theories and models to that of objects and events	Evolution to construct the relation between absence of force in a model and absence of interaction between objects: i.e., if no forces are exerted between the systems representing two objects, then the objects can neither be acting on one another through contact nor repelling or attracting one another at a distance.
Type 4	Relations within the world of theories and models	Evolution from an everyday explanation of uniform motion to the physics theory/model: i.e., from the idea that motion with fixed speed and direction requires force to be exerted in the direction of motion, to the idea that motion with constant velocity needs no force. Evolution to construct the relation between force and change of velocity.

relations, usually starting with Type 1 and concluding with Type 4.

Activity focused on Type 1 relations aims to help students move toward descriptions of objects and events that will help to establish the perspective taken by physics theory; often, but not necessarily, this requires that students develop their way of "viewing" a situation from everyday to physics knowledge, modifying their description accordingly. Such activities are normally rather rare in physics teaching at the secondary school level: When probed, teachers suggest that Type 1 relations are obvious because students only have to look at the situation in order to see the relation (Tiberghien et al., in press). However, examining Type 1 relations between objects and events allows the designer to emphasize the role of perception in physics learning, particularly the requirement to shape what is perceived. This emphasis is based on an epistemology of science that recognizes the importance of perception and observation (Hacking, 1983/2005) and on a social constructivist psychology in which the mediating function of language plays a central part (Tiberghien et al., in press). Activities addressing Type 1 relations aim to help students become aware of different events, some of which are spontaneously taken into account, others of which are brought to students' attention from the way in which the activity itself is designed. The idea is that students become able to "see events differently," as well as becoming able to use a relevant physical language to describe them. For example, students become able to describe two *static* objects that are in contact with each other as an *event* involving *action* (each object *acts on* the other in all situations, irrespective of whether there is motion). In everyday discourse, of course, static objects in contact with each other do not constitute an event. This part of the design ensures that a relevant language of description for objects and events is taught. This is Wittgenstein's famous idea of "seeing as"; these activities help students to view objects and events in the terms of a physicist (Sensevy, Tiberghien, Santini, Laube, & Griggs, 2008). The other types of relation will be illustrated and discussed in the example of a teaching activity that follows.

Example of a teaching activity informed by the use of the Lyon design tools. These design tools have been used to develop teaching sequences in various topics of physics at secondary school level (optics, energy, electricity, etc.; PEGASE, 2008). For example, the following teaching activity involves three types of relation. Within the teaching sequence concerned, it takes place after the Newtonian model of motion has been introduced to students and they have had some time to develop understanding of it. Prior to this activity, the students have also directly experienced the objects and events to which it relates, by throwing and catching a heavy medicine ball. This has made them conscious not just of the action of their hands on the ball, but of the action of the ball on their hands, and of the associated motions of their hands. Now the students are presented with two theoretical representations of the forces modeling the upward flight of the ball once it has left their hands (see Figure 3). They are told that one of these diagrams corresponds to an intuitive explanation of the situation that shares with the theory developed by Aristotle

The activity starts from a short text describing, in everyday language, the "intuitive" and Newtonian accounts of the motion of an object. These diagrams are then presented as representations of the forces acting on a medicine ball (MB) after it has been thrown and while it is still moving upward:

The following question is then posed to students: With the help of information given at the beginning of this activity, identify which diagram (A or B) is analyzing the situation intuitively. Justify your choice.

FIGURE 3. *Outline of task intended to lead students to compare two models through relating their components to a particular material situation.*

the idea that there must always be a force acting in the direction of movement of the medicine ball, whereas the other diagram corresponds to the later theory of motion that rejects this idea; the theory initiated by Galileo and formalized by Newton. The students' task is to decide which diagram corresponds to which theory, in line with the designers' aim to develop an activity that leads students to differentiate explicitly between the two models, so focusing on what the Modeling Relations tool would class as a Type 4 relation. However, in relating these models to their direct experience with the ball, it is also necessary for students to give attention to Type 2 and Type 3 relations from objects/events to theories/models and the reverse. A focus on these types of relation involves explicit attention to modeling processes, even if these are already implicit in the physics description. From the intuitive perspective no distinction is made between the phase where the hand is touching the ball and the phase once the ball has left the hand: Thus the hand is "seen" to act upon the ball, leading to inclusion of an upward force in the direction of motion, as well as the downward gravitational force exerted by attraction to the Earth and the downward drag force exerted by contact with the surrounding air while moving upward (as shown in Diagram A of Figure 3). From the Newtonian perspective these two phases are "seen" as distinct: Thus during the (second) phase of upward motion that is being modeled, only the downward gravitational and drag forces are operative (as shown in Diagram B of Figure 3).

The design anticipates that such an activity will be carried out in small groups where students, through interaction, will construct their point of view, followed at the whole-class level by critical review of the accounts that students have developed, leading to institutionalization of the situation in terms of recognized scientific knowledge (in accordance with the grand theory of social constructivism).

This teaching sequence has been evaluated in terms of students' learning, through quasi-experimental comparison employing pre- to posttest analysis of 10 classes against 9 classes of similar students following the school's usual teaching approach. In quantitative terms, students who had followed the designed teaching sequence performed significantly (in statistical terms) better across a broad range of questions. Furthermore, in a qualitative comparison between 1 class from each treatment, such students were better able to relate physics concepts with material situations and to relate everyday and physics language (Tiberghien & Malkoun, 2007).

A Social Constructivist Intermediate Framework for Design (Leeds Group)

Like the Lyon group, the Leeds group draws upon Vygotskian grand theory on meaning making. Perspectives on personal sense making and a realist ontology (e.g., Harré, 1988) have been integrated with this grand theory to develop a social constructivist perspective on learning scientific concepts in formal settings (Driver, Asoko, Leach, Mortimer, & Scott, 1994; Leach & Scott, 2003). This is an intermediate framework with the purpose of theorizing the process whereby learners are introduced to, and become able to use, scientific ideas that already exist on the social plane, and in doing so reconcile those ideas with familiar everyday ways of making sense of the material world.

What differentiates the intermediate framework employed by the Leeds group most clearly from that of the Lyon group is the stronger accent on social constructivist grand theory of learning, rather than on epistemological theory. A related difference is its use of the Bakhtinian notion of *social language*. As Wertsch (1991, p. 46) has pointed out, the Vygotskian view is limited in that there is no recognition of the *different* forms of intermental functioning that occur on the social plane. Bakhtin (1934/1981) draws attention to the fact that different modes of discourse are used in different parts of society, and he refers to these as "social languages" that "may be juxtaposed to one another, mutually supplement one another and co-exist in the consciousness of real people" (p. 292). In particular, scientific knowledge is developed as a result of social interactions between scientists who work together to build explanations for evidence about the physical world. Ideas like *momentum* are developed because scientists agree upon a particular way of modeling some aspect of the physical world. The *scientific* social language, then, is that developed within the scientific community and based on the use of specific concepts such as energy, mass, and entropy. It involves the development of models that provide an account of phenomena in the natural world, and it is characterized by certain key epistemological features such as the development of theories that can be generally applied to different phenomena and situations.

It is worth being clear that Bakhtin's term *social language* is used to incorporate a variety of semiotic practices other than language. As well as discourse, mathematical and visual representations are central to the social language of science. Furthermore, the term *social language* should not be taken as indicating that the social language of science is "just talk"; it should, in principle, be consistent with empirical evidence about the material world. Scientists are not in a position to create their social language in isolation from empirical data.

Equally, learners are constantly exposed to an *everyday* social language. Everyday conceptions are developed through social interactions: Children grow up surrounded by talk that suggests that things "burn away to nothing" and pictures where a balloon filled with air is drawn "floating upwards" on a string rather than sinking downward toward the floor. If learning science is conceptualized as "learning to talk in new ways" or "learning to talk science" (Lemke, 1990), then it may appear deceptively straightforward: simply a matter of learning to talk about familiar phenomena in new ways. However, some aspects of scientific social language are known to be strikingly difficult for learners to use and understand. The notion of internalization is used to address this issue. Individual learners must reconstruct the sense of the talk and activities that surround them on the social plane, reorganizing their existing ideas and ways of thinking accordingly.

The social constructivist intermediate framework brings together the social-interactive and personal-sense-making parts of the learning process and identifies language as the central form of mediational means on both social and personal planes. It draws upon sociocultural approaches in conceptualizing learning in terms of developing a new social language and in identifying epistemological differences between social languages. It draws upon evidence about alternative conceptions in clarifying the nature of the learning required by students in order to make personal interpretations of the social language of science.

We will now present two tools that have been developed from this specific framework in order to inform decisions about the design of science teaching.

The Learning Demand tool. Learning Demand (Leach & Scott, 2002) was developed to help to identify the conceptual aims of science teaching at a fine grain size. It draws directly upon the intermediate framework sketched above in that it involves making a comparison between two social languages, namely, the social language of school science and the social language that school students are likely to use when discussing phenomena and events at a given point in their science education. The conceptual basis of explanations of the material world in each social language is identified in terms of ontology, epistemology, and the patterns of reasoning on which explanations are based. These are then compared between the two social languages; learning demands are identified in terms of the different concepts and associated ontology, epistemology, and patterns of reasoning used in students' everyday social language and the social language of school science. The learning demands in a given area of content are then used to identify the precise nature of the content-specific learning that needs to be supported through teaching. The aims of the Learning Demand tool therefore have much in common with the Knowledge Distance tool (although curriculum knowledge to be taught is treated differently by each tool).

We will now illustrate the identification of learning demands by considering teaching in the lower secondary school that addresses the behavior of simple electrical circuits. Prior to teaching, students' explanations tend to be based on the behavior of "electricity," whereas the physics explanations to be taught are based on current, charge, and energy: The two social languages have a different ontology. Furthermore, students tend to draw upon different explanations for different circuits, whereas the social language of physics is based on the use of a single explanatory framework: The two social languages have a different epistemology. In addition, students' explanations tend to be based on a linear causal sequence of events, starting in the battery with events in the resistive components of the circuit following later. By contrast, the social language of school physics describes circuits as integrated systems where events happen at the same time. The social languages of students and school physics are thus based on different patterns of reasoning.

The Communicative Approach tool. Communicative Approach (Mortimer & Scott, 2003) is a design tool that focuses on classroom discourse and provides a perspective on *how* the teacher interacts with students to develop ideas on the social plane of the classroom. The verbal communication in the classroom is described in terms of two dimensions: authoritative/dialogic and interactive/noninteractive. In authoritative discourse, an authority figure (normally the teacher) controls the direction of the talk, to focus it on one point of view (normally the scientific view). In dialogic discourse, the discourse is open to different points of view, both everyday and scientific. Interactive talk involves more than one speaker, whereas noninteractive talk involves just one speaker. Mortimer and Scott relate the communicative approach to different teaching purposes. For example, authoritative talk is more appropriate when new meanings are being introduced on the social plane of the classroom, whereas dialogic talk is more appropriate when students' everyday views are being explored. One would therefore expect to see shifts between authoritative and dialogic discourse throughout a sequence of lessons, according to the purpose of the talk (Scott, Mortimer, & Aguiar, 2006).

Example of a teaching activity informed by the use of the Leeds design tools. The following teaching activity was developed for use with students age 11 to 12 at the beginning of lower secondary school. It is designed to address a learning demand about the *patterns of reasoning* used to explain the behavior of electric circuits. The physics explanation introduced to students in the lower secondary school is based on a view of electric circuits as integrated systems; by contrast, students' explanations typically describe a linear sequence of causal events. The teaching activity begins with an activity called the BIG Circuit. Prior to the lesson, the teacher constructs a simple series circuit with a power source at one end of the teaching room, a bulb at the other end of the teaching room, and very long wires stretched around the walls of the room connecting the circuit together. Pupils are asked to predict what will happen when the circuit is connected. Many will say that *there will be a short delay* between connecting the circuit and the bulb lighting. This is because they believe that *the electricity will take a short time* to travel from the power source to the bulb. However, when the circuit is connected there is no delay: The lamp lights instantaneously. This raises a question to be addressed in subsequent teaching: *How come the bulb in the BIG Circuit lights instantaneously?* At this stage, the communicative approach built into the design of the teaching is dialogic because the teaching purpose is probing and elaborating meanings.

The next stage of the activity sees the teacher introducing an analogy to the class. The particular analogy involves bread vans (i.e., charges) delivering bread (i.e., energy) from a bakery (i.e., power source) to a supermarket (i.e., bulb). The bread vans are in a nose-to-tail loop all the way from the bakery to the supermarket so that when one bread van stops, all have to stop, and when one moves, all move. This explains why as soon as the bread vans move (i.e., the circuit is connected), bread can be delivered (i.e., the bulb can light). The activity begins to develop students' understanding of the ontology of scientific explanations of the behavior of simple electric circuits by introducing, and differentiating between, charge, current, and energy. It introduces a particular analogy and uses it systematically to bridge between the observed behavior of an electric circuit and the target physics explanation. At this stage, the communicative approach built into the design of the teaching becomes more authoritative in that the teaching purpose is to introduce new content.

The teaching sequence has been evaluated in terms of students' learning. Using a pre-post test design, the construction of explanations based on the taught model of electric circuits was examined in 15 classes of students (age 11–12) who had followed the designed teaching sequence and was compared with 15 classes of similar students who had followed the school's usual teaching approach. In all cases, students who had followed the designed teaching were significantly better at generating explanations (Leach, Hind, Lewis, & Scott, 2006).

Conclusion

Rather than presenting and critiquing any single research program, the purpose of this article is to illustrate how intermediate frameworks and design tools can be used in the design process to mediate between grand theory and the practice of science and mathematics teaching. However, we should also draw attention to some important difficulties inherent in designing teaching sequences in this way and in making evidence-based claims about the design process. A first, and perhaps obvious, point to make is that designed teaching sequences draw upon significant influences beyond intermediate frameworks and design tools. In crafting teaching sequences, designers bring to bear their experience about learners' and teachers' expectations of "good lessons," the availability of resources, institutional constraints, and so on. Such insights are critical to the success of the teaching and are not directly attributable to intermediate frameworks or design tools. Furthermore, there is an essential, creative step between using design tools and creating activities such as the puzzle enlargement or the BIG Circuit—and then to setting such activities into a whole sequence that is coherent in its aims. We know from our own experience as designers that using design tools does not always result in the development of "neat" teaching activities such as those described in this article—and furthermore that it is possible to develop such "neat" teaching activities without the use of design tools. However, the use of design tools brings into focus the design problem to be addressed and provides a mechanism for evaluating the effectiveness of teaching activities in addressing that design problem.

This article has highlighted how intermediate frameworks and design tools serve to organize the contribution of grand theories to the process of designing and evaluating teaching sequences by extracting relevant components of the theories and coordinating and contextualizing their application. However, we should highlight differences in the salience of grand theories within the intermediate frameworks that we have presented. In the case of the intermediate TDS, it could be said that although its initial development was "scaffolded" by epistemological and cognitive grand theory, the role of that grand theory has "faded" with the establishment of a robust apparatus and discourse more closely tailored to the needs of didactical design. However, in the case of the Two Worlds and Social Constructivist intermediate frameworks, reference to grand theory remains more active and explicit, even if there is a similar potential for more independent use and development of the intermediate frameworks and their associated design tools.

The cognate examples of the Two Worlds and Social Constructivist frameworks raise further important issues. On the one hand, they point to the potential for a proliferation of intermediate frameworks drawing on the same (or similar) grand theories. On the other hand, they raise the question of whether such intermediate frameworks might be able to offer each other commensurable and complementary design tools. For example, the Communicative Approach tool associated with the Social Constructivist framework offers more strongly theorized guidance on an issue that the Two Worlds framework acknowledges: namely, the organization of classroom discussion through an initial phase of discussion between students in groups, followed by a phase of class discussion in which the teacher plays a stronger organizing role, leading to a final phase of institutionalization in which the teacher makes explicit the distinctive terms in which the scientific community formulates knowledge of the phenomena in question. Equally, the Modeling Relations tool associated with the Two Worlds framework provides a specific form of guidance to design teaching activities according to the types of knowledge they introduce at a fine-grained analysis of what the Social Constructivist framework conceptualizes in terms of relations between social languages.

Given the theoretical commonalities between the two frameworks, there is clearly potential for the Communicative Approach and Modeling Relations tools to be released from their current anchorings and taken up within the other intermediate framework. Moreover, although the "borrowed" tool might initially be treated as a simple complement to those associated with the existing framework, in time it can be expected to become more integrally appropriated into that framework. From the resulting re-equilibration of the two intermediate frameworks, a productive synthesis of the parallel Knowledge Distance and Learning Demand tools might develop, opening the way to the two frameworks becoming fused. Such tool exchange between intermediate frameworks can promote their integration, acting as a counterbalance to proliferation. On the other hand, the very contrast in terminology between *Knowledge* Distance and *Learning* Demand signals the different weight that these tools give to grand theories of epistemology of science (greater in the first) and social constructivism (greater in the second). Thus different cultural emphases and values can act against synthesis of intermediate frameworks and promote a proliferation of design tools. Indeed, we suspect that such differences, associated with the contrasts we noted earlier in the institutional conditions under which designers and researchers work, have inhibited interaction between North American and European traditions of design research, despite their grounding in similar grand theories of learning.

Not all design tools, however, are strongly theorized. For example, although originating with the intermediate TDS, the Didactical Variables tool carries no immediate theoretical presumptions (although it may become associated with some deriving from paradigmatic examples of the tool in use). Effectively, then, it can be used compatibly with any intermediate framework that incorporates a concern with the ways in which student response may be sensitive to task characteristics. Although it gains power through use in conjunction with some theorized framework to carry out prior analysis of a teaching situation, it can also be used more pragmatically in the light of pedagogical intuitions or of empirical findings arising from posterior analysis of implementation of a teaching design. Moreover, such pragmatically guided use of the Didactical Variables tool can complement theoretically guided use, serving as a useful mechanism to identify critical features falling outside the scope of the theoretical framework, which would otherwise go unrecognized. Thus it provides a general heuristic as useful to "design as implementation" through iterative cycles of revision as to "design as intention."

Our interest in building a public apparatus of design tools has concerns similar to those of the project to establish a database of design principles (Kali, 2008). Both reflect a movement toward establishing collective knowledge to underpin more explicit design approaches, including the communal debating, refining, and warranting of such knowledge (Kelly, Baek, Lesh, & Bannand-Ritland, 2008). Kali notes how difficult it is to extract the design thinking that resides in traditional forms of publication for the purpose of creating new designs and suggests that new approaches are required for the explication, organization, and synthesis of such knowledge. Comparing the tools that we have described in this article with Kali's closest equivalent—her system of principles—the main differences of approach appear to be in the salient role of theory in many of the tools that we have described and their emphasis more on sensitizing the designer to crucial issues than on specifying particular courses of action.

Equally, although it is important to recognize differences between, on the one hand, teaching sequences that aim to support fine-grained evolution in students' grasp of particular conceptual systems within mathematics and science, often over relatively short time spans and, on the other, learning environments that aim to support longer term development of students' engagement in broader disciplinary practices, these are not incompatible educational aims and processes but interdependent ones (Sfard, 1998). Hence we see value in dialogue between different traditions of design research with the goal of developing a more comprehensive perspective from which apparently diverse intermediate frameworks and design tools for mathematics and science education can be refined and employed in a more coordinated manner.

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